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Design of Missile Guidance Law via Variable Structure Control

Jongki Moon,* Kiseok Kim,* and Youdan Kim†
Seoul National University, Seoul 151-742, Republic of Korea

The missile guidance law utilizing variable structure control is proposed. The acceleration command input is determined considering the target acceleration as an uncertainty. The proposed guidance law uses only the information for the target acceleration bound; therefore, the precise measuring of target acceleration during the maneuver is not required, and the robustness to the target maneuver is achieved. It is also shown that the proposed guidance law can be classified into the augmented true proportional navigation or the augmented realistic true proportional navigation guidance law. Numerical simulations show that the proposed guidance law yields better performance compared to existing guidance laws.

I. Introduction

PROPORTIONAL navigation guidance (PNG) was first developed during the 1950s, and during the 1970s and 1980s various PNG laws such as pure proportional navigation (PPN), true proportional navigation (TPN), the optimal guidance law (OGL), generalized TPN (GTPN), and realistic TPN (RTPN) have been developed.[1,2] Lots of studies have been performed to obtain analytical solutions as well as to analyze the capture regions of the guidance laws. With the development of accurate avionic sensors, augmented proportional navigation (APN) and the predictive guidance law (PGL) utilizing the information about target acceleration were also proposed. By analyzing the various guidance laws, the characteristics of the guidance laws, capture regions, and pursuing performance were compared.[1,2]

During the last decade, various practical problems have been considered for real implementation.[3–9] Guidance laws have been extended to the three-dimensional engagement case for highly maneuverable targets[3] and have been applied to missile systems considering the induced drag and time-varying velocity[4] and the internal dynamics of missiles.[5] Studies on the time-to-go estimation,[6] ideal proportional navigation,[7] and OGLs subject to various constraints were also performed.[8,9] New guidance laws using nonlinear control theories such as the Lyapunov function method,[10] the geometric control theory,[11] and the nonlinear H_∞ control theory[12] have also been developed.

The variable structure control law, or the sliding mode control law, usually designs a sliding surface to satisfy the design objective.[13–15] The sliding mode control is similar to the backstepping control when considering the dynamic structure and the distinctive two-loop controller design. When the backstepping controller is used, the stability of each loop as well as the overall two-loop stability must be assessed. However, in the case where the sliding mode control is used, the overall stability is always guaranteed if the state variables can reach the sliding surface in a finite time.[14,15] In this paper the concept of equivalent control[16–18] is used to derive the sliding mode control law. The equivalent control consists of terms for the known dynamics and for the uncertain parameters.

The variable structure control law has been applied to many guidance problems. Zhou et al. proposed an adaptive sliding mode guidance law using linearized equations.[19] Babu et al. studied the guidance law for highly maneuvering targets using the sliding surface of the zero line-of-sight (LOS) rate based on the Lyapunov method.[20] Brierly and Longchamp studied the sliding mode guidance law including rigid-body dynamics and actuator time delay.[21]

In this paper a new sliding mode guidance law based on the nonlinear planar engagement kinematics is proposed. The only assumption of the proposed guidance law is that the information for the maximum target acceleration is available. Therefore, the proposed guidance law is robust with respect to target maneuver. Various sliding surfaces are suggested to design the guidance law, and the decreasing boundary layer scheme is introduced. Through the proper choice of the sliding surfaces, it is shown that the proposed guidance law demands less maximum control effort and less intercept time. We derive the guidance law including general function terms so that different guidance laws can be designed by specifying the general function terms.

The paper is organized as follows. In Sec. II the variable structure control theory is briefly summarized. Section III formulates the missile-target engagement problem and proposes a new guidance law. Numerical simulation results are shown in Sec. IV, and conclusions are reported in Sec. V.

II. Variable Structure Control

Consider a nonlinear system[14]

$$\dot{\eta} = f_1(\eta, \xi) + \delta_\eta(\eta, \xi) \tag{1}$$

$$\dot{\xi} = f_a(\eta, \xi) + G_a(\eta, \xi)[u + \delta_\xi(\eta, \xi, u)] \tag{2}$$

where δ_ξ is a matched uncertainty and δ_η is an unmatched uncertainty. Note that δ_ξ satisfies the matching condition, and therefore it affects a system in the same way as the control input u does.

Now, let us introduce a backstepping approach to design a sliding surface $\xi = \phi(\eta)$. First, assume that ξ in Eq. (1) is a virtual control input, and design $\xi = \phi(\eta)$ such that the following system has the desired properties:

$$\dot{\eta} = f_1[\eta, \phi(\eta)] + \delta_\eta[\eta, \phi(\eta)] \tag{3}$$

To do this, a new variable z is introduced:

$$z = \xi - \phi(\eta) \tag{4}$$

If $z = 0$, then $\xi = \phi(\eta)$ is obtained, and finally a variable η has the desired properties. Variable structure control makes z equal to zero in finite time and then maintains the condition $z = 0$ for all future time. That is, variable structure control makes $z = 0$ be a positive invariant set of the closed-loop system. A typical trajectory under variable structure control consists of a reaching mode, during which the sliding variable z moves toward the sliding surface $z = 0$, and a sliding mode, during which the sliding variable is confined to the sliding surface $z = 0$.

Differentiating Eq. (4) with respect to time and substituting Eqs. (2) and (3) into the resulting equation yield the following equation:

$$\dot{z} = f_a(\eta, \xi) + G_a(\eta, \xi)[u + \delta_\xi(\eta, \xi, u)] - \frac{\partial \phi}{\partial \eta}[f_1(\eta, \xi) + \delta_\eta(\eta, \xi)] \tag{5}$$

Let us take the control input as

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*Graduate Student, Department of Aerospace Engineering.
†Associate Professor, Department of Aerospace Engineering; ydkim@snu.ac.kr. Senior Member AIAA.

$$u = u_{eq} + G_a(\eta, \xi)^{-1}\nu \tag{6}$$

where u_{eq} is determined to cancel the known terms on the right-hand side of Eq. (5) as follows:

$$u_{eq} = G_a(\eta, \xi)^{-1}\left[-f_a(\eta, \xi) + \frac{\partial\phi}{\partial\eta} f_1(\eta, \xi)\right] \tag{7}$$

If there is no uncertainty in the system, then $u = u_{eq}$ is the equivalent control that yields the closed-loop motion in the sliding manifold once the system state vector is in the manifold (4), $z = 0$. In this study, because the system includes uncertainties, u_{eq} is an estimate of a known portion of equivalent control.

Now, to design ν, let us consider the case where uncertainties exist. Substituting Eq. (7) into Eq. (5) gives

$$\dot{z} = \nu + \Delta(\eta, \xi, \nu) \tag{8}$$

where

$$\Delta(\eta, \xi, \nu) = G_a(\eta, \xi)\delta_\xi\left[\eta, \xi, u_{eq} + G_a(\eta, \xi)^{-1}\nu\right] - \frac{\partial\phi}{\partial\eta}\delta_\eta(\eta, \xi) \tag{9}$$

Assume that Δ of Eq. (9) satisfies the following inequality:

$$\|\Delta(\eta, \xi, \nu)\|_\infty \le \rho(\eta, \xi) + d\|\nu\|_\infty \tag{10}$$

where the continuous function $\rho(\eta, \xi) \ge 0$ and the constant $d \in [0, 1]$ are known. Using Eq. (10), the perturbed control ν can be designed. Equation (8) can be rewritten as follows:

$$\dot{z}_i = \nu_i + \Delta_i(\eta, \xi, \nu), \qquad i = 1, \ldots, p \tag{11}$$

Let us take ν_i as follows:

$$\nu_i = -\frac{\beta(\eta, \xi)}{1 - d}\,\mathrm{sgn}(z_i), \qquad i = 1, \ldots, p \tag{12}$$

where $\mathrm{sgn}(\cdot)$ denotes the signum function, and

$$\beta(\eta, \xi) \ge \rho(\eta, \xi) + b, \qquad b > 0 \tag{13}$$

Consider a Lyapunov function candidate $V = \sum_{i=1}^{p} \frac{1}{2} z_i^2$ for Eq. (11). Differentiating V with respect to time and substituting Eq. (10) and Eq. (12) into the resulting equation gives

$$\dot{V} = \sum_{i=1}^{p} \dot{V}_i = \sum_{i=1}^{p} z_i\dot{z}_i = \sum_{i=1}^{p}[z_i\nu_i + z_i\Delta_i(\eta, \xi, \nu)]$$
$$\le \sum_{i=1}^{p}\left[-\frac{\beta(\eta, \xi)}{1-d}|z_i| + \rho(\eta, \xi)|z_i| + d\frac{\beta(\eta, \xi)}{1-d}|z_i|\right]$$
$$= \sum_{i=1}^{p}[-\beta(\eta, \xi)|z_i| + \rho(\eta, \xi)|z_i|] \le \sum_{i=1}^{p} -b|z_i| \tag{14}$$

It is certain from Eq. (14) that the trajectory of z converges to the manifold $z = 0$ at some time, and it will be confined to that manifold for all future time. The sliding mode controller contains the discontinuous nonlinearity $\mathrm{sgn}(z_i)$. Nonlinearity of the control input can cause chattering as a result of delays or imperfections in the switching devices. To eliminate the chattering, the following continuous approximation of the perturbed control ν can be used:

$$\nu_i = -\frac{\beta(\eta, \xi)}{1 - d}\,\mathrm{sat}_\varepsilon(z_i), \qquad \varepsilon > 0 \tag{15}$$

where $\mathrm{sat}_\varepsilon(\cdot)$ is the saturation function, which is defined by the following:

$$\mathrm{sat}_\varepsilon(z_i) = \begin{cases} z_i/\varepsilon, & |z_i| \le \varepsilon \\ 1, & |z_i| > \varepsilon \end{cases} \tag{16}$$

When Eq. (15) is used in Eq. (11) instead of Eq. (12), and (14) is satisfied only in the region of $|z_i| > \varepsilon$, and η is confined to a positive invariant set restricted by the functions of ε (Ref. 15).

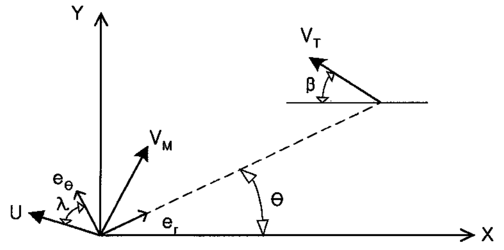


Fig. 1 Missile target engagement geometry.

III. Application of Variable Structure Control to Missile Guidance Problem

A. Problem Formulation

Consider a two-dimensional air-to-air engagement as shown in Fig. 1, where a missile is attempting to intercept a moving target. The missile and target are both assumed as point masses. The dynamic equations are given by[12]

$$\ddot{r} - r\dot{\theta}^2 = a_{T,r} - a_{M,r}, \qquad r\ddot{\theta} + 2\dot{r}\dot{\theta} = a_{T,\theta} - a_{M,\theta} \tag{17}$$

where r denotes the relative distance between the missile and the target, θ denotes the LOS angle, $a_{T,r}$ and $a_{T,\theta}$ are the radial and tangential components of target acceleration, and $a_{M,r}$ and $a_{M,\theta}$ are the radial and tangential components of missile acceleration, respectively. Note from Eq. (17) that $a_{T,r}$ and $a_{T,\theta}$ can be treated as uncertainties of the system.

Let us introduce state variables $[V_r \;\; V_\theta] = [\dot{r} \;\; r\dot{\theta}]$ and rewrite Eq. (17) in the state-space form as follows:

$$\frac{d}{dt}\begin{bmatrix} r \\ V_r \\ V_\theta \end{bmatrix} = \begin{bmatrix} V_r \\ V_\theta^2/r \\ -V_r V_\theta/r \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} w - \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} u \tag{18}$$

where

$$w = \begin{bmatrix} w_r \\ w_\theta \end{bmatrix} = \begin{bmatrix} a_{T,r} \\ a_{T,\theta} \end{bmatrix}$$

is a disturbance vector and

$$u = \begin{bmatrix} u_r \\ u_\theta \end{bmatrix} = \begin{bmatrix} a_{M,r} \\ a_{M,\theta} \end{bmatrix}$$

is a control input vector, respectively. Equation (18) can be rewritten as

$$\dot{r} = V_r \tag{19}$$

$$\dot{V}_r = V_\theta^2/r + w_r - u_r \tag{20}$$

$$\dot{V}_\theta = -V_r V_\theta/r + w_\theta - u_\theta \tag{21}$$

Comparing Eqs. (19–21) with Eqs. (1) and (2) gives the following relations:

$$\eta = r, \qquad \xi = [V_r \quad V_\theta] \tag{22}$$

$$f_1 = \xi_1, \qquad f_a = \begin{bmatrix} \xi_2^2/\eta \\ -\xi_1\xi_2/\eta \end{bmatrix}, \qquad G_a = -\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{23}$$

$$\delta_\eta = 0, \qquad \delta_\xi = -\begin{bmatrix} w_r \\ w_\theta \end{bmatrix} \tag{24}$$

Several remarks can be made for the preceding two-dimensional missile guidance problem:

1) Equation (19) shows that this problem does not have unmatched uncertainty.

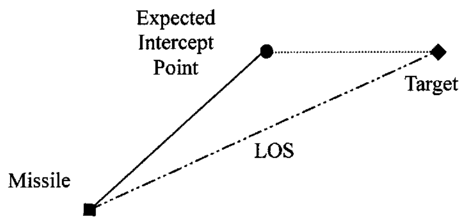


Fig. 2 Collision triangle.

2) Equations (20) and (21) show that the uncertainty w satisfies the matching condition, and for a nonmaneuvering target, i.e., $w = 0$, the system becomes nominal.

3) In Eq. (20) the control input cannot be designed like u_θ because u_r is the guidance command related to the direction of LOS. Thus, u_r is designed so that V_r tracks the reference approaching velocity. In this paper both systems when u_r is available and not available are considered in the numerical simulation.

4) For the interception the relative speed between the missile and the target must satisfy the following necessary condition:

Proposition 1. There exists $\tau \geq 0$ such that $V_r(t) \leq -\delta < 0$ for $t \geq \tau, t \in [\tau \;\; t_f]$.

B. Sliding Surface Design

In this section the guidance law will be designed using the variable structure control scheme. The sliding surfaces should be designed properly to satisfy the design objective, which is a critical aspect of missile guidance law design using variable structure control.

1. Design of $V_\theta = \phi_2(r)$

Let us design the sliding surface $V_\theta = \phi_2(r)$. Note that $\phi_2(r)$ does not affect the relative distance r significantly, and therefore $\phi_2(r)$ can be designed without considering the design of $\phi_1(r)$. Consider an engagement of the missile and the target represented in Fig. 2. It is desirable to design the sliding surface $\phi_2(r)$ so that the guidance command can be applied to make the missile be in the collision triangle. The conception that the LOS rate should be zero for interception was adopted in many other PNG laws. In this case the sliding surface is chosen as follows:

$$\phi_2(r) = 0 \tag{25}$$

When the sliding mode is chosen as Eq. (25), there is a possibility that an abrupt large guidance command is generated during the initial maneuver. To avoid the abrupt large guidance command, monotonically decreasing $\phi_2(r)$ can be selected such that $\phi_2(r)$ and $\dot{\phi}_2(r)$ approach zero as r goes to zero. The following are possible candidates for the sliding surfaces that satisfy the preceding condition:

$$\phi_2(r) = V_\theta(0)\left[\frac{r(0)}{r}\right]^{-\alpha}, \qquad \alpha > 0 \tag{26}$$

$$\phi_2(r) = \frac{V_\theta(0)}{\exp(-1)}\exp\left[-\frac{r(0)}{r}\right] \tag{27}$$

2. Design of $V_r = \phi_1(r)$

Strictly speaking, $V_r = \phi_1(r)$ is not a sliding surface because $V_r = \phi_1(r)$ does not have to depend on the state variables. In this study $V_r = \phi_1(r)$ is regarded as the sliding surface, because this is more convenient in designing a control law. The following are possible candidates of $\phi_1(r)$:

$$\phi_1(r) = V_f \tag{28}$$

$$\phi_1(r) = V(0) + [V_f - V(0)][1 - r/r(0)] \tag{29}$$

where V_f is the desired final relative velocity between the missile and the target. Good tracking performance is not required for this sliding surface because the engagement is guaranteed as long as [Proposition 1] is satisfied. Thus, $V_r = \phi_1(r)$ is not critical in the missile guidance law design.

C. Guidance Law Design

In this section the guidance law is designed using variable structure control for the system represented in Eqs. (19–21).

1. Guidance Law for Control Variable u_θ

Consider a guidance law for the control variable u_θ. Using Eq. (6), the guidance law is obtained as

$$u_\theta = u_{\theta,\mathrm{eq}} - \nu_\theta \tag{30}$$

An equivalent guidance command for the known dynamics $u_{\theta,\mathrm{eq}}$ is obtained from the known system parameters. Substituting Eqs. (22) and (23) into Eq. (7) yields

$$u_{\theta,\mathrm{eq}} = -\frac{V_r V_\theta}{r} - \frac{\mathrm{d}\phi_2}{\mathrm{d}r}V_r \tag{31}$$

The remaining perturbed guidance command can be determined through the following procedure. First, substituting Eq. (22) into Eq. (9) yields the following uncertainty term:

$$\Delta_2 = w_\theta \tag{32}$$

It is natural that the system uncertainty is caused by target acceleration because the target acceleration cannot be measured with precision and without time delay. Assume that the upper bound of the target acceleration can be estimated as

$$|w_\theta(t)|_\infty \leq k_2 \tag{33}$$

Comparing Eq. (33) with Eq. (10) yields $\rho = k_2$ and $d = 0$. Substituting $\rho = k_2$ into Eq. (13) , we have

$$\beta(r, V_r, V_\theta) > k_2 \tag{34}$$

Assume that the function $\beta(r, V_r, V_\theta)$ has the following form to satisfy Eq. (34):

$$\beta = a_2 + g(r, V_r, V_\theta) \tag{35}$$

where $a_2 > k_2$ and $g(r, V_r, V_\theta) \geq 0$. There are no restrictions on $g(r, V_r, V_\theta)$ except $g(r, V_r, V_\theta) \geq 0$; therefore, any positive definite function $g(r, V_r, V_\theta)$ can be used. This means that a different selection of $g(r, V_r, V_\theta)$ yields a different guidance law; therefore, several guidance laws can be derived from the design of $g(r, V_r, V_\theta)$. Substituting Eq. (35) into Eq. (15) with $d = 0$ yields the following:

$$\nu_\theta = -[a_2 + g(r, V_r, V_\theta)]\,\mathrm{sgn}[V_\theta - \phi_2(r)] \tag{36}$$

Using Eqs. (30), (31), and (36), the guidance law is obtained as

$$u_\theta = -\frac{V_r V_\theta}{r} - \frac{\mathrm{d}\phi_2}{\mathrm{d}r}V_r + [a_2 + g(r, V_r, V_\theta)]\,\mathrm{sgn}[V_\theta - \phi_2(r)] \tag{37}$$

The reachability to the sliding surface in a finite time can be examined by substituting Eq. (37) into Eq. (21):

$$\dot{V}_\theta - \frac{\mathrm{d}\phi_2}{\mathrm{d}r}V_r = \frac{\mathrm{d}}{\mathrm{d}t}[V_\theta - \phi_2]$$
$$= w_\theta - [a_2 + g(r, V_r, V_\theta)]\,\mathrm{sgn}[V_\theta - \phi_2(r)] \tag{38}$$

From the condition that $\beta = a_2 + g(r, V_r, V_\theta) > k_2 \geq |w_\theta|$, the velocity V_θ reaches the sliding surface $V_\theta = \phi_2(r)$ in a finite time.

When the sliding mode control scheme is used, the chattering problem must be considered with much care. To solve the chattering problem, usually the function sat(·) is used for sgn(·):

$$u_\theta = -\frac{V_r V_\theta}{r} - \frac{\mathrm{d}\phi_2}{\mathrm{d}r}V_r + [a_2 + g(r, V_r, V_\theta)]\,\mathrm{sat}_\varepsilon[V_\theta - \phi_2(r)] \tag{39}$$

The function sat(·) is defined in Eq. (16). Replacing sat(·) for sgn(·) causes another problem in the stability within the boundary layer. For this reason small gain was used for the sliding surface in Ref. 13. However, because of the choice of small gain it did not confirm the reachability to the boundary layer of the sliding surface. In this study, by Eq. (38) the reachability to the boundary layer of the sliding surface is confirmed. However, in the case of a highly maneuvering target the proposed guidance law yields a high-gain guidance law, which may cause rapid chattering and bad noise characteristics within the boundary layer. To overcome these problems, the decreasing boundary-layer scheme is introduced. That is, ε in Eq. (39) is chosen as decreasing with the decreasing of the distance between the missile and the target. For example, the following defines a linearly decreasing ε:

$$\varepsilon = [r/r(0)]\varepsilon_0 + [1 - r/r(0)]\varepsilon_f \tag{40}$$

where ε_0 and ε_f denote the initial and final value of ε, respectively.

2. Comparison with Other Guidance Laws

In this section comparisons of Eq. (37) with other guidance laws are discussed. Substituting Eq. (25) into Eq. (37) yields the following:

$$u_\theta = -(V_r V_\theta / r) + g(r, V_r, V_\theta)\,\mathrm{sgn}[V_\theta] + a_2\,\mathrm{sgn}[V_\theta] \tag{41}$$

For the first case let us take $g(r, V_r, V_\theta)$ as follows:

$$g(r, V_r, V_\theta) = -N(V_r/r)|V_\theta| \tag{42}$$

By substituting Eq. (42) into Eq. (41) with $\mathrm{sgn}[V_\theta] = V_\theta/|V_\theta|$, guidance command u_θ is given as

$$u_\theta = -(N+1)(V_r V_\theta/r) + a_2\,\mathrm{sgn}[V_\theta] \tag{43}$$

where $N+1$ is a navigation constant. The first term of Eq. (43) represents the realistic TPN guidance law; therefore, Eq. (43) can be classified into the augmented RTPN guidance law.

Now, under the assumption that $NV_r(0) < V_r$, let us take $g(r, V_r, V_\theta) = -\{N[V_r(0)/r] - V_r/r\}|V_\theta|$, which yields the following guidance law:

$$u_\theta = -N[V_r(0)V_\theta/r] + a_2\,\mathrm{sgn}[V_\theta] \tag{44}$$

Similarly to Eq. (43), the first term of Eq. (44) represents the TPN guidance law, thus Eq. (44) can be classified into the augmented TPN guidance law. In Ref. 15 Eq. (44) is named as the simplified adaptive sliding mode guidance (ASMG) law.

3. Guidance Law for Control Variable u_r

When the control input u_r is available, the guidance law for control variable u_r can be designed as follows. From Eq. (6) the guidance law is represented as

$$u_r = u_{r,\mathrm{eq}} - \nu_r \tag{45}$$

For simple derivation we select $\phi_1(r) = V_f$. Then the equivalent guidance command is obtained by substituting Eqs. (22) and (23) into Eq. (7) as follows:

$$u_{r,\mathrm{eq}} = V_\theta^2 \big/ r \tag{46}$$

The uncertainty term is not considered as was the case in u_θ because the control variable u_r cannot be controlled arbitrarily. Thus, the guidance law is designed as follows:

$$u_r = V_\theta^2 \big/ r + a_1\,\mathrm{sat}_\varepsilon[V_r - V_f] \tag{47}$$

where the constant a_1 is a control gain. Good tracking performance of V_r is not required because the engagement is guaranteed if Proposition 1 is satisfied during the maneuver time. Thus, the design for control variable u_r is not critical in the missile guidance law using the variable structure control method.

IV. Numerical Simulation

Numerical simulations are performed to investigate the performance of the proposed guidance law. It is assumed that the guidance command is not limited. In engagement case 1 the proposed guidance law is compared with the conventional ideal proportional navigation (IPN) represented by the following guidance command:

$$u_r = N_{\mathrm{IPN}} V_\theta^2 \big/ r, \qquad u_\theta = -N_{\mathrm{IPN}} V_r V_\theta / r \tag{48}$$

In engagement case 2 the simplified ASMG law[15] is compared.

$$u_\theta = -N_a V_r(0) V_\theta / r + c\,\mathrm{sat}_{\varepsilon_2}[V_\theta] \tag{49}$$

In both cases random noise with average power 0.5 g is added in the target acceleration, and a first-order filter with time constant 0.3 s is considered for the acceleration estimation.

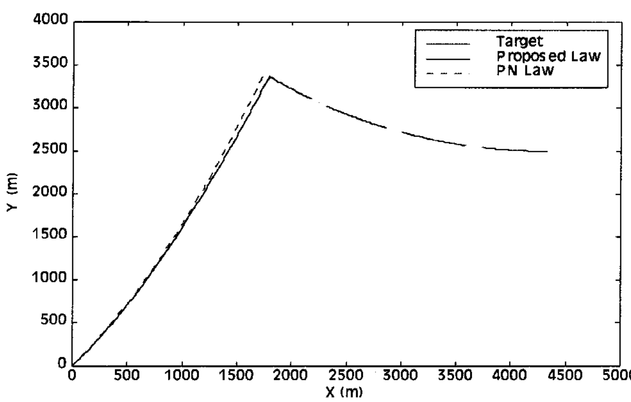
A. Engagement Case 1

In case 1 the PNG law in Eq. (48) is compared with the proposed guidance law. The initial relative distance is 5 km, the closing velocity is 700 m/s, and the lateral relative velocity is 30 m/s. We consider the situation in which the target is initially flying at 350 m/s and maneuvers with 3-g acceleration. Thus, k_2 in Eq. (33) is 3-g(m/s^2). For the missile the function $g(r, V_r, V_\theta)$ in Eq. (39) is adopted as follows:

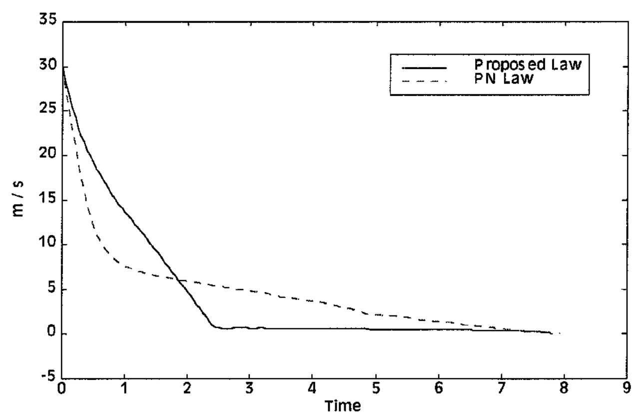
$$g(r, V_r, V_\theta) = -N(V_r/r) \tag{50}$$

The constant N is chosen as 3, the constant a_2 in Eq. (37) is set to be 30, and $\varepsilon = 1$ is selected. In Eq. (47) the reference reaching velocity and control gain are chosen as $V_f = -710$ m/s and $a_1 = 3$, respectively. For fair comparison the navigation constant N_{IPN} in Eq. (48) is chosen as 22. The maximum acceleration in IPN law is assumed to be limited to 50 m/s^2.

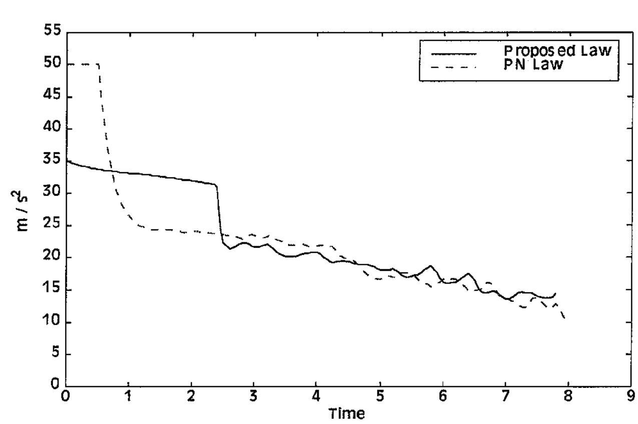
Figures 3a–3c show the trajectories of the missile and the target, the relative lateral velocity V_θ, and the guidance command, respectively. The interception time is 7.82 s for the proposed law



a) Trajectory

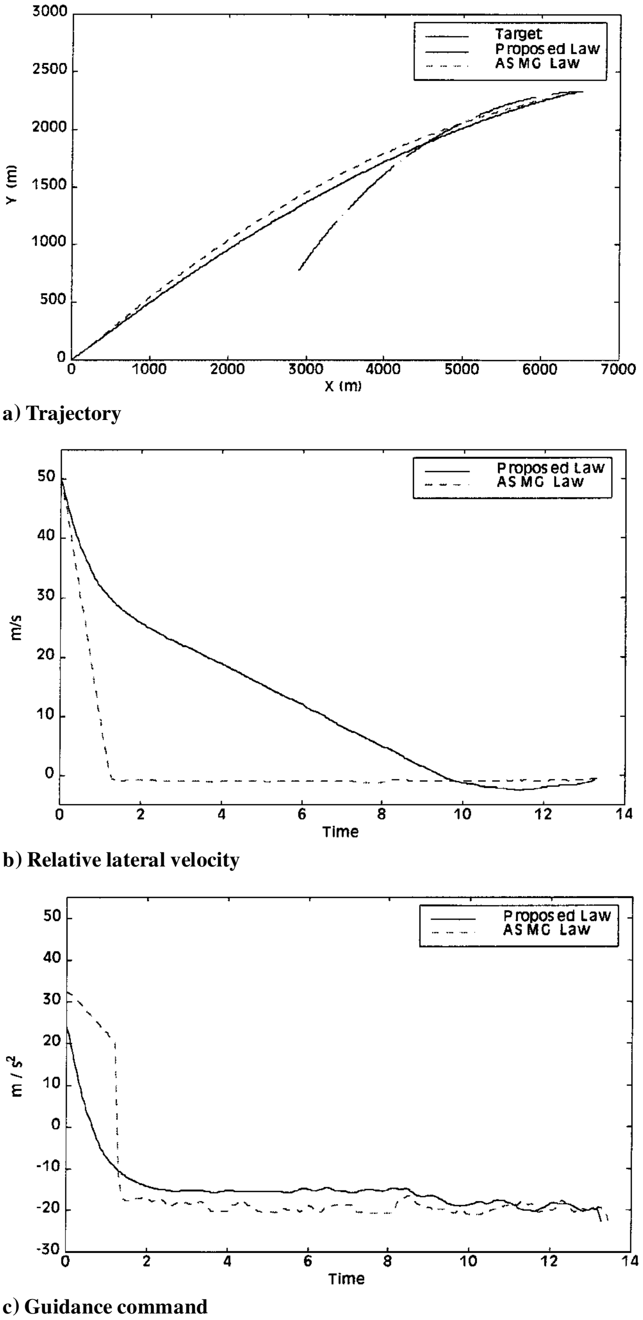


b) Relative lateral velocity



c) Guidance command

Fig. 3 Engagement case 1.



a) Trajectory

b) Relative lateral velocity

c) Guidance command

Fig. 4 Engagement case 2.

and 8.00 s for the IPN law. In Fig. 3b, because of high gain during the initial engagement, both the proposed guidance law and the IPN law try to reduce the relative lateral velocity. After the relative lateral velocity is reduced to about 5 m/s, the IPN law reduces V_θ gradually; however, the proposed law continuously reduces V_θ until V_θ reaches to near zero. Figure 3c demonstrates that the maximum magnitude of the proposed guidance law is less than that of the PN law, while the noise characteristics are similar.

B. Engagement Case 2

In case 2 the simplified ASMG law in Eq. (49) is compared with the proposed guidance law. It is assumed that the control variable u_r is unavailable, or $u_r = 0$. The initial closing velocity is 300 m/s, the initial lateral velocity is 50 m/s, and the relative distance is 3 km. The moving direction of the target is 135 deg right to that of case 1, and the maximum acceleration of the target is 2 g, or $k_2 = 20$.

For the proposed guidance law $a_2 = 20$ is chosen, and ε is described by Eq. (40) with $\varepsilon_0 = 5$ and $\varepsilon_f = 1$, that is, the decreasing boundary-layer scheme is adopted. The sliding surface is defined by Eq. (27) for gradually nullifying LOS rate. The function $g(r, V_r, V_\theta)$ is not used, or $g(r, V_r, V_\theta) = 0$. For the ASMG law $N_a = 4$, $c = 20$, and $\varepsilon_2 = 1$ in Eq. (49) are chosen for fair comparison.

Figure 4a shows the trajectories of the missile and the target, Fig. 4b the relative lateral velocity V_θ, and Fig. 4c the guidance command. The interception time is 13.25 s for the proposed law and 13.55 s for the ASMG law. Figure 4b shows that the proposed guidance law reduces the relative lateral velocity V_θ gradually, whereas the high gain ASMG law reduces V_θ rapidly. Figure 4c shows that the proposed guidance law requires less control effort than ASMG law, and its noise characteristics are a little bit better than that of the ASMG law.

V. Conclusions

A systematic approach to design a new guidance law is proposed using variable structure control. The guidance command is derived based on the nonlinear planar engagement kinematics, and the target acceleration is treated as disturbance. The proposed law requires only the target acceleration limit, and therefore the exact information of target acceleration is not necessary. By using this approach, the robustness of the guidance law with respect to the target maneuver can be achieved. The relations between the proposed law and other conventional proportional navigation laws are also discussed. Two-type sliding surfaces, rapid zeroing the LOS rate surface, and gradual zeroing LOS rate surface are introduced. For a high-gain sliding mode control law the decreasing boundary-layer scheme is introduced. Numerical simulations show that the proposed guidance law is effective in the sense that the maximum magnitude of the guidance command and intercept time are smaller than those of other guidance laws.

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